A Comparison of PID Controller Tuning Methods

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The derivative mode is often omitted in PID control strategies because it proves difficult to arrive by trial-and-error at a set of constants which meet plant requirements. The primary objective of this paper was to evaluate several model-based PID tuning methods. For lag-dominant processes, it was recommended that the SIMC algorithm first be employed to determine whether satisfactory performance can be obtained with PI control. If it cannot, then derivative action should be introduced using the DS-d technique. For delay-dominant systems, IMC tuning is preferred. It was observed that when configured with the same derivative filter factor, the series form of the PID controller produces smoother valve adjustments than the parallel version, at the expense of a slight decrease in best achievable performance. Increasing this parameter improves the control effort but limits achievable performance.

Le mode différentiel est souvent omis dans les stratégies de contrôle PID parce qu'il s'avère difficile de parvenir par essais-erreurs à un ensemble de constantes qui répondent aux attentes de l'usine. Le principal objectif de cet article est d'évaluer plusieurs méthodes de réglage PID basées sur des modèles. Pour les procédés où dominent les retards, il est recommandé d'employer d'abord l'algorithme SIMC pour déterminer si une performance satisfaisante peut être obtenue par le contrôle PI. Dans la négative, une action par différentiation devrait alors être introduite au moyen de la technique DS-d. Pour les systèmes où dominent les retards, le réglage IMC est préférable. On a observé que pour une configuration utilisant le même facteur filtre différentiel, la disposition en série du contrôleur PID entraîne des ajustements de vannes plus souples que la version en parallèle, au détriment d'une légère perte dans la recherche de la meilleure performance possible. L'augmentation de ce paramètre améliore l'effort de contrôle mais limite la performance réalisable.

Keywords: PID control, tuning, stability, controller performance, robustness

It is a remarkable fact that the vast majority of chemical processes are well-controlled with the PID algorithm or one of its simpler forms. A widely-accepted rule-of-thumb is that use of the derivative mode should be confined to applications characterized by long time delays and/or higher-order dynamic lags, and then only provided that the feedback signal is virtually free of measurement noise. According to Luyben and Luyben (1997), approximately 80% of industrial controllers are PI or P-only, with PID accounting for the remaining 20%. Other studies (e.g. Bialkowski, 1992) report a much lower PID implementation rate.

As one would expect, introduction of a third degree-of-freedom into the design problem enables the engineer to obtain tighter control. This is because the phase lead contributed by the derivative term reduces the impact of deadtime and other sources of phase lag which limit the achievable performance of the feedback loop. From a state space point of view, the derivative of the process variable (*PV*) can be regarded as an estimate of a system state. For second-order processes with no zeros or deadtime, knowledge of the *PV* and its derivative permits one to assign the closed-loop poles to arbitrary locations (Kailath, 1980). Perhaps less well-recognized is the fact that derivative action normally increases the robustness of a control system to high-frequency modelling errors. Unfortunately, these improvements are obtained at the cost of larger control effort, which may lead to increased wear-and-tear and frequent saturation of the final control element, greater interaction with other control loops and operator distress (Harris and Tyreus, 1987).

It is an established fact that first-order-plus-deadtime (FOPDT) approximations can adequately explain the behaviour of a wide range of processes and many industrially-proven techniques are now available for fitting these models to plant data (see Seborg et al., 2004). The main objective of this paper was to compare

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several well-known PID tuning strategies when applied to the control of FOPDT systems. (A comprehensive survey of model-based PI and PID tuning rules has been published by O'Dwyer (2003).) Methods proposed by Rivera et al. (1986), Lee et al. (1998), Chen and Seborg (2002) and Skogestad (2003) were considered because they reduce the complexity of the PID tuning problem to specification of a single adjustable parameter. This facilitates a trade-off between performance on the one hand versus control effort and robustness on the other. The work was further motivated by a desire to address the following questions in the context of FOPDT processes:

- When is the use of derivative action beneficial?
- Which tuning algorithm is least sensitive to measurement noise?
- What are the practical differences between the series and parallel representations of the PID control law?

The tuning formulae associated with each technique will be stated and analyzed by comparing their integral time expressions. A series of simulation experiments are subsequently described which illustrate the control effort and robustness of each algorithm, tuned to provide equal performance when implemented on a digital controller. Guidelines are then developed to assist the user in selecting the most appropriate strategy for a particular application.

DEFINITIONS

Figure 1 is a schematic diagram of a feedback control loop. The symbols r and Y represent the setpoint and process variable, respectively. The disturbance d enters at the process input and

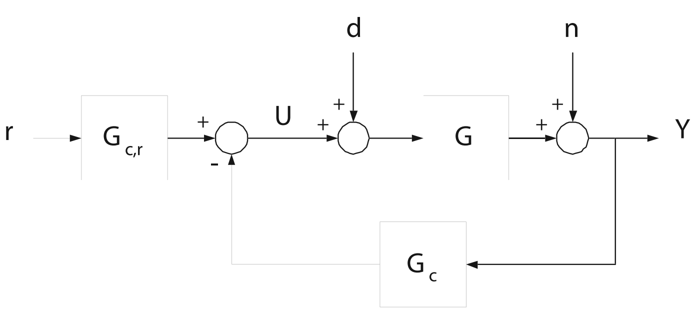

Figure 1. Feedback control loop

the feedback signal is corrupted by random measurement noise n. The controller output is denoted by U. It is assumed that the combined dynamics of the process, final control element and sensor are well-approximated by the FOPDT transfer function:

$$G(s) = \frac{K e^{-Ds}}{\tau s + 1} \tag{1}$$

The usual practise is to implement derivative on measurement only, in order to avoid "derivative kick" on setpoint changes. Hence the feedback controller may be represented as

$$G_C(s) = K_C \left(1 + \frac{1}{\tau_I s} + \tau_D s \right) \tag{2}$$

and the servo compensator as

$$G_{C,r}(s) = K_C \left(1 + \frac{1}{\tau_I s} \right) \tag{3}$$

The PID controller transfer function (2) is not physically realizable since no device can be constructed which truly differentiates an input signal (Luyben and Luyben, 1997). A number of approximations of the derivative term have consequently been developed over the years by the vendors. These include the "parallel" or "non-interacting" form:

$$G_{C,1}(s) = K_C \left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha \tau_D s + 1} \right) \tag{4}$$

as well as the "series" or "interacting" controller:

$$G_{C,2}(s) = K'_C \left(1 + \frac{1}{\tau'_I s} \right) \frac{(\tau'_D s + 1)}{(\alpha \tau'_D s + 1)} \tag{5}$$

(see Kaya and Scheib, 1988). The parameter α will be referred to here as the *derivative filter factor*. Most control systems provide the user with limited flexibility in the choice of α (e.g. $\alpha \in [0.05,\ 0.2]$) but in some instances it is pre-set to a low value such as 1/16 or 0.1. A third alternative places the "textbook" controller (2) in series with a first-order filter:

$$G_{C,3}(s) = \left[K_C \left(1 + \frac{1}{\tau_I s} + \tau_D s \right) \right] \frac{1}{\tau_F s + 1} \tag{6}$$

Strictly speaking, this version of the PID controller is not available in many commercial systems. It may be approximated by configuring a *PV* filter with time constant τ_F and replacing the term in square brackets with Equation (4) (or (5), if it can be factored into series form). α should be set to its minimum value when attempting to implement $G_{C,3}(s)$.

There are numerous accepted mathematical definitions of performance, robustness and control effort. In this report, performance was quantified as the ratio of the variance of the control error σ_e^2 to $\sigma_{e,M}^2$, the minimum achievable *using a parallel PID structure with* $\alpha = 0.05$:

$$Performance = \frac{\sigma_{e,MV}^2}{\sigma_e^2} \times 100\,\% \tag{7}$$

The control error $e \equiv r - Y$, the deviation of the process variable from setpoint. This performance benchmark was selected since for a given value of α, the parallel PID controller can be tuned for a smaller error variance than is possible using the series form (see Simulation Results) and because the minimum achievable error variance decreases with α. The performance of a PID controller, series or parallel, is then bounded by 0 (unstable) and 100% (best achievable). As explained by MacGregor et al. (1984), $\sigma_{e,MV}^2$, may be interpreted as the mean square error instead of the error variance if, e.g., one is interested in modelling the input disturbance as a series of deterministic step changes rather than a random walk stochastic process.

Control effort was measured by reference to the variance of the valve adjustments exhibited by the minimum variance PID controller, $\sigma_{\Delta U,MV}^2$:

$$Control\ \ Effort = \frac{\sigma_{\Delta U}^2}{\sigma_{\Delta U,\,MV}^2} \times 100\,\% \tag{8}$$

Notice that it is possible for the control effort of a given loop to exceed 100%. This would simply indicate that the variance of its control moves is even greater than that required by the parallel

PID controller which minimizes the variance of the error. A small control effort is obviously desirable since it implies smooth increments in the final control element (e.g. valve).

The robust stability of each controller was assessed by simultaneously increasing the steady-state gain (K_0) time constant (τ_0) and deadtime (D_0) of the *actual* plant until one or more poles of the discrete-time characteristic equation were found to lie outside the unit circle. Robustness was defined as the largest deviation, $\delta_{max,}$ for which closed-loop stability was maintained, where

$$\delta \equiv \frac{K_0 - K}{K} \times 100\% = \frac{\tau_0 - \tau}{\tau} \times 100\% = \frac{D_0 - D}{D} \times 100\% \qquad (9)$$

The idea of simulating modelling errors by perturbing the FOPDT parameters in this manner was taken from Marlin (1995). The effect is analogous to decreasing throughput in a continuous stirred-tank heater. For instance, it is apparent from a dynamic energy balance that if the feed flowrate were to decrease by half, the gain, time constant and transport delay of the process transfer function would double, resulting in the multiplicative parameter error $\delta = 100\%$ Foley et al. (2005) found that δ_{max} is strongly correlated with the relative delay margin (Wang and Cluett, 2000).

MODEL-BASED PID TUNING METHODS

This section presents the tuning formulae obtained when the aforementioned methods are applied to FOPDT plants. Each of these techniques could be classified as “Direct Synthesis” (Seborg et al., 2004) as it is intended to produce a desired closed-loop transfer function. The algorithms may be distinguished primarily according to the design response (servo vs. regulatory) and the means employed to reduce the associated high-order controller to PID form. The first four attempt to generate the FOPDT servo transfer function

$$\frac{Y(s)}{r(s)} = \frac{e^{-Ds}}{\lambda s + 1} \qquad (10)$$

where λ is the user-specified closed-loop time constant. Using Equation (1) and Figure 1 (taking $G_{C,r}(s) = G_C(s)$) it may be shown that the required feedback strategy is

$$G_C^*(s) = \frac{(\tau s + 1)/K}{\lambda s + 1 - e^{-Ds}} \qquad (11)$$

This can be regarded as Internal Model Control (Garcia and Morari, 1982) or, equivalently, a Smith PI control scheme (Smith, 1957) with $K_C = \tau/(K\lambda)$ and $\tau_I = \tau$. Since many control systems do not provide Smith Predictor or Internal Model Control functionality it was of interest to develop tuning procedures whereby (11) could be mimicked using the standard PID algorithm.

IMCF (Table 1, Row F of Rivera et al., 1986)

After introducing a first-order Padé approximation for the delay term in Equation (1), Rivera and co-workers demonstrated that the PID compensator (2) with

$$K_C = \frac{\tau + (D/2)}{K(\lambda + D/2)} \qquad (12)$$

$$\tau_I = \tau + (D/2) \qquad (13)$$

$$\tau_D = \frac{\tau D/2}{\tau + D/2} \qquad (14)$$

produces the closed-loop servo transfer function

$$\frac{Y(s)}{r(s)} \approx \frac{1-(D/2)s}{\lambda s + 1} \approx \frac{e^{-(D/2)s}}{\lambda s + 1} \qquad (15)$$

Rivera et al. (1986) suggested that λ be chosen greater than *max* (0.1τ, $0.8D$). Decreasing λ generally improves performance at the cost of greater control effort and reduced robustness. An important advantage of the IMC approach is that the controller zeros may be factored as

$$G_C(s) = \frac{K_C(\tau s + 1)\,[(D/2)s + 1]}{\tau_I\, s} \qquad (16)$$

thus the IMC PID tuning rules can always be converted for series implementation (Harris and Tyreus, 1987; Chien and Fruehauf, 1990).

LEE (Equation (22) of Lee et al., 1998)

Lee et al. (1998) asserted that a better approximation of $G_C^*(s)$ is obtained through Maclaurin series expansion of $sG_C^*(s)$. Truncating the series after three terms yields

$$K_C = \frac{\tau_I}{K(\lambda + D)} \qquad (17)$$

$$\tau_I = \tau + \frac{D^2}{2(\lambda + D)} \qquad (18)$$

$$\tau_D = \frac{D^2}{6(\lambda + D)}\left(3 - \frac{D}{\tau_I}\right) \qquad (19)$$

An alternative approach leading to a controller with similar time-domain characteristics was described by Wang et al. (1995). Their tuning rules were also derived by representing $sG_C^*(s)$ as a second-order polynomial but its coefficients were fit by evaluating $sG_C^*(s)$ at two frequencies. The authors made the useful observation that τ/λ can be interpreted as the desired initial change in controller output, expressed as a fraction of its eventual response, following a step change in setpoint. Wang et al. (2001) proposed a technique for determining the PID transfer function which lies closest to IMC in the frequency range of interest through numerical minimization of a quadratic cost index.

SIMC (Skogestad, 2003)

If e^{-Ds} is approximated with the first-order Maclaurin series $1 - Ds$, $G_C^*(s)$ reduces to a PI compensator with

$$K_C = \frac{\tau}{K(\lambda + D)} \qquad (20)$$

and $\tau_I = \tau$. A feedback controller configured in this way can be tuned for excellent servo behaviour but its regulatory response may be unacceptably sluggish for lag-dominant processes, i.e., when the deadtime-to-time constant ratio $D/\tau << 1$. When high-performance regulatory control of such a process is required, SIMC offers the flexibility of choosing $\tau_I < \tau$ in order to place greater emphasis on eliminating low-frequency disturbances. The SIMC tuning method requires that the proportional gain be computed as (20) and the integral time as the lesser of $\tau_{I,1}$ and $\tau_{I,2}$ where

$$\tau_{I,1} = \tau \tag{21}$$

$$\tau_{I,2} = 4(\lambda + D) \tag{22}$$

Skogestad (2003) recommended that derivative action not be applied to FOPDT plants, thus

$$\tau_D = 0 \tag{23}$$

The particular form of Equation (22) was designed to produce a critically-damped response for a pure integrator, which can be viewed as an extreme example of a lag-dominant system.

IMCG (Table 1, Row G of Rivera et al., 1986)

When a first-order Padé approximation is instead employed in Equation (11), it is easily demonstrated that $G_C^*(S)$ assumes the structure (6) with parameters:

$$K_C = \frac{\tau + (D/2)}{K(\lambda + D)} \tag{24}$$

$$\tau_I = \tau + (D/2) \tag{25}$$

$$\tau_D = \frac{\tau D/2}{\tau + D/2} \tag{26}$$

$$\tau_F = \frac{\lambda D/2}{\lambda + D} \tag{27}$$

Notice that as λ is increased, the filter time constant $\tau_F \rightarrow D/2$, which implies that the PV filter $1/(\tau_F s + 1)$ in Equation (6) will cancel a controller zero. In other words, as the IMCG strategy is detuned,

$$G_{C,3}(s) = \frac{K_C(\tau s + 1)\,[(D/2)s + 1]}{\tau_I\, s(\tau_F\, s + 1)} \rightarrow \frac{\tau}{K(\lambda + D)} \cdot \frac{(\tau s + 1)}{\tau s} \tag{28}$$

which is the SIMC PI controller (20)-(21). Luyben (2001) compared the Ciancone-Marlin (Marlin, 1995) and Ziegler-Nichols tuning methods with the IMCG algorithm configured for high performance control ($\lambda = max(0.2\tau,\ 0.25D)$ of FOPDT processes. It was concluded that IMCG provides superior rejection of input disturbances and is less sensitive to measurement noise and errors in the process model.

DS-d (Chen and Seborg, 2002)

It may be argued that there is a logical contradiction inherent in designing PID controllers to achieve a desired servo response when derivative action is not normally applied to the command signal. The DS-d (Direct Synthesis for Disturbance Rejection) method was instead derived with the aim of producing the *regulatory* closed loop transfer function

$$\frac{Y(s)}{d(s)} = \frac{(\tau_I / K_C)[1 + (D/2)s]\,s\,e^{-Ds}}{(\lambda s + 1)^3} \tag{29}$$

The practical advantage is that the factor $(\tau s + 1)$ is no longer present in the denominator of $Y(s)/d(s)$ permitting tight regulation of lag-dominant systems.

The required PID controller parameters are obtained from:

$$K_C = \frac{\left(2\tau D + \frac{D^2}{2}\right)\left(3\lambda + \frac{D}{2}\right) - 2\lambda^3 - 3\lambda^2 D}{2K\left(\lambda + \frac{D}{2}\right)^3} \tag{30}$$

$$\tau_I = \frac{\left(2\tau D + \frac{D^2}{2}\right)\left(3\lambda + \frac{D}{2}\right) - 2\lambda^3 - 3\lambda^2 D}{(2\tau + D)D} \tag{31}$$

$$\tau_D = \frac{3\lambda^2 \tau D + \frac{\tau D^2}{2}\left(3\lambda + \frac{D}{2}\right) - 2(\tau + D)\lambda^3}{\left(2\tau D + \frac{D^2}{2}\right)\left(3\lambda + \frac{D}{2}\right) - 2\lambda^3 - 3\lambda^2 D} \tag{32}$$

As was true of the preceding techniques, an increase in λ yields a slower closed-loop response (lower performance). However, there is a limit to the extent to which the DS-d controller can be detuned: one must restrict λ in order that the controller parameters remain positive.

It should be mentioned that Lee et al. (1998) also developed tuning rules for the regulatory case. The settings stated as Equation (32) of their paper give a regulatory response which approximates

$$\frac{Y(s)}{d(s)} = \frac{K e^{-Ds}}{(\tau s + 1)} \, \frac{[(\lambda s + 1)^2 - e^{-Ds}(\gamma s + 1)]}{(\lambda s + 1)^2} \tag{33}$$

The variable γ is chosen such that the term in square brackets contains a zero to cancel the plant pole at $s = -1/\tau$.

DISCUSSION

By comparison of Equations (18) and (13) it is apparent that the integral time of the LEE algorithm is less than that of IMCF for all $\lambda > 0$. Equation (18) also implies that $\tau_I^{LEE} > \tau$. Recognizing that $\tau_I^{SIMC} \leq \tau$ leads to the conclusion

$$\tau_I^{SIMC} < \tau_I^{LEE} < \tau_I^{IMCF} \tag{34}$$

(The IMCG algorithm has been omitted from this analysis since it is a four-parameter design but it may be recalled that IMCG coincides with SIMC for large λ.) When tuned for the same level of performance, it is expected that the controller with the largest reset time τ_I would compensate by employing the strongest proportional and/or derivative action. One would therefore anticipate that an IMCF PID controller would prove more noise-sensitive than the others, i.e., possess the largest control effort, because its high-frequency gain is greatest. In terms of the time-domain dynamics, a PID strategy tuned according to the IMCF rule will exhibit the strongest initial response to disturbances.

The remaining error will be integrated to zero most slowly because this scheme generally assigns least penalty to low-frequency deviations.

No general statements may be made with respect to the DS-d integral time in relation to those of the LEE and SIMC controllers. However, Equation (31) can be written in terms of the dimensionless parameters $\Lambda \equiv \lambda/\tau$ and $\psi \equiv D/\tau$ as

$$\frac{\tau_I}{\tau} = \frac{\left(2\Psi + \frac{\Psi^2}{2}\right)\left(3\Lambda + \frac{\Psi}{2}\right) - 2\Lambda^3 - 3\Lambda^2\Psi}{(2+\Psi)\Psi} \tag{35}$$

Taking the partial derivative of Equation (35) with respect to Λ and equating the result to zero, it is found that for a given ψ, the DS-d integral time is maximized by the choice

$$\Lambda = \frac{\Psi}{2}\left[-1 + \sqrt{2 + \frac{4}{\Psi}}\right] \tag{36}$$

It may be verified by plotting $\tau_{I,\,max}/\tau$ versus Ψ, substituting Λ in Equation (35) with Equation (36), that $\tau_{I,max}/\tau \le 1 + \Psi/2$ for all Ψ. This illustrates that $\tau_I^{DS\text{-}d} \le \tau_I^{IMCF}$ for all λ, D and τ, from which one would infer that IMCF will typically require larger valve adjustments when configured for the same error variance as the DS-d regulator.

SIMULATION RESULTS

The main purpose of this section is to compare the DS-d, SIMC, IMCF, IMCG and LEE PID tuning methods on the bases of control effort, robustness and achievable *regulatory* performance. The servo response is naturally of secondary importance in most applications where one is considering the use of derivative action since this mode applies to the *PV* only. Hence the setpoint signal in Figure 1 was held constant ($r(s) = 0$) while the input disturbance was modelled as a random walk.

The simulations were carried out in discrete-time with a modified z-transform representation of the plant dynamics (see Palmor and Shinnar, 1979). Controllers (4) and (5) were discretized via the backward difference approximation $s \approx (1 - z^{-1})/T$ where T is the controller execution period (Åström and Hägglund, 1995). This resulted in the expressions

$$G_{C,i}(z^{-1}) = \frac{K_D\,(c_0 + c_1 z^{-1} + c_2 z^{-2})}{1 - z^{-1}} \cdot \frac{(1-f)}{(1 - f z^{-1})} \cdot \frac{(1-g)}{(1 - g z^{-1})}$$

and

$$i = 1: \quad c_0 = 1 + \frac{T}{\tau_I} + \frac{\tau_D}{T} + \frac{\alpha\tau_D}{T}\left(1 + \frac{T}{\tau_I}\right), \quad c_1 = -\left\{1 + \frac{\tau_D}{T}\left[2 + \alpha\left(2 + \frac{T}{\tau_I}\right)\right]\right\} \tag{37}$$

$$c_2 = \frac{\tau_D(1+\alpha)}{T}, \quad K_D = K_C, \quad f = \frac{\alpha\tau_D}{\alpha\tau_D + T}, \quad g = 0$$

$$i = 2: \quad c_0 = \left(1 + \frac{T}{\tau'_I}\right)\left(1 + \frac{\tau'_D}{T}\right), \quad c_1 = -\left[1 + \frac{2\tau'_D}{T} + \frac{\tau'_D}{\tau'_I}\right], \quad c_2 = \frac{\tau'_D}{T},$$

$$K_D = K'_C, \quad f = \frac{\alpha\tau'_D}{\alpha\tau'_D + T}, \quad g = 0$$

The IMCG strategy $G_{C.3}(z^{-1})$ was programmed using the preceding formulae with $\alpha = 0.5$ and $g = \tau_F/(\tau_F + T)$.

In order to compensate for the sampling delay when calculating the controller parameters, the process deadtime was raised to $D + T/2$ as suggested by Franklin et al. (1990). Closed-loop variances were obtained by numerical evaluation of the following contour integrals

$$\sigma_e^2 \equiv Var(e_t) = \frac{1}{2\pi j}\oint_{|z|=1} \frac{|G(z^{-1})|^2\sigma_a^2 + |1 - z^{-1}|^2\sigma_n^2}{|1 + G_C(z^{-1})G(z^{-1})|^2\,|1 - z^{-1}|^2}\,\frac{dz}{z} \tag{38}$$

$$\sigma_{\Delta U}^2 \equiv Var(\Delta U_t) = \frac{1}{2\pi j}\oint_{|z|=1} \frac{|G_C(z^{-1})|^2\left[\,|G(z^{-1})|^2_a\sigma^2 + |1 - z^{-1}|^2\sigma_n^2\right]}{|1 + G_C(z^{-1})G(z^{-1})|^2}\,\frac{dz}{z} \tag{39}$$

using the computer algorithm given in Chapter 5 of Åström (1970). (Alternatively, one might employ the *norm* function of the Matlab® Control Systems Toolbox.) The symbol σ_n^2 denotes the variance of the measurement noise n_t and σ_a^2 that of the differenced disturbance Δd_t. The behaviour of a continuous FOPDT system operating under model-based PID control depends only upon the parameters λ/τ, D/τ, α and $\sigma_n^2/(K^2\sigma_a^2)$. According to Marlin (1995), the closed-loop response of a digital PI controller will closely resemble that of an identically-tuned analog system when the sampling/control interval T is chosen less than $0.05\ (D + \tau)$. In the simulation studies which follow, T was specified as $0.03\ (D + \tau)$ and σ_a^2 was left at unity.

Figures 2 and 3 illustrate the performance, control effort and robustness of the five tuning methods for regulatory control of the FOPDT process

$$G(s) = \frac{e^{-s}}{10s + 1} \tag{40}$$

The control effort (variance of changes in the manipulated variable) generally increases in Figure 2a and robustness decreases (Figure 2b) as performance improves. To facilitate the comparison, the tuning guidelines reported earlier for the IMC algorithms were not utilized. Instead, λ was reduced until the best achievable performance was attained, i.e., such that further decreases would cause performance to degrade. Points to the left of the diamond marker on the SIMC curve were generated using the integral time (21) and points to the right with (22). Sample calculations may be found in the Appendix.

It is evident from Figure 2 that DS-d provides for excellent regulation of lag-dominant processes in the absence of measurement noise. For all performance levels greater than 20%, the DS-d

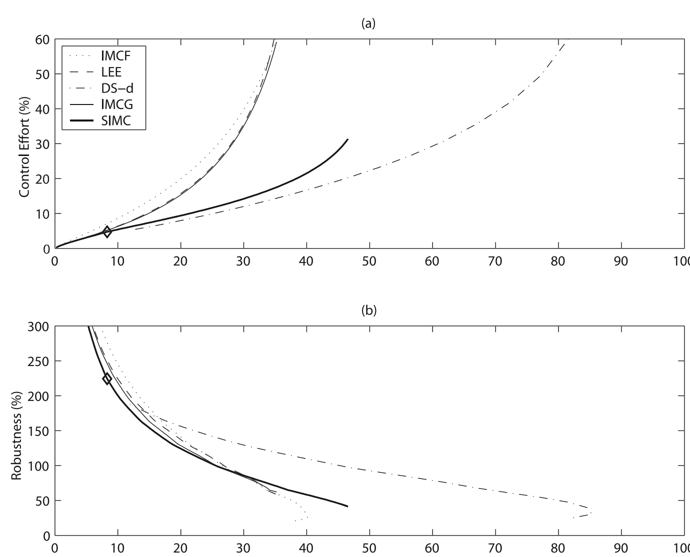

Figure 2. Parallel PID control of lag-dominant plant ($D/\tau = 0.1$, $\sigma_n^2 = 0$, $\alpha = 0.1$)

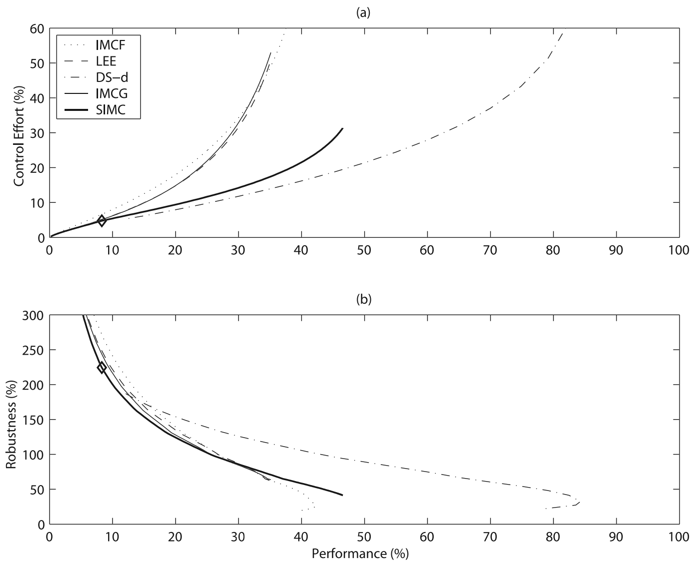

Figure 3. Series PID control of lag-dominant plant ($D/\tau = 0.1$, $\sigma_n^2 = 0$, $\alpha = 0.1$)

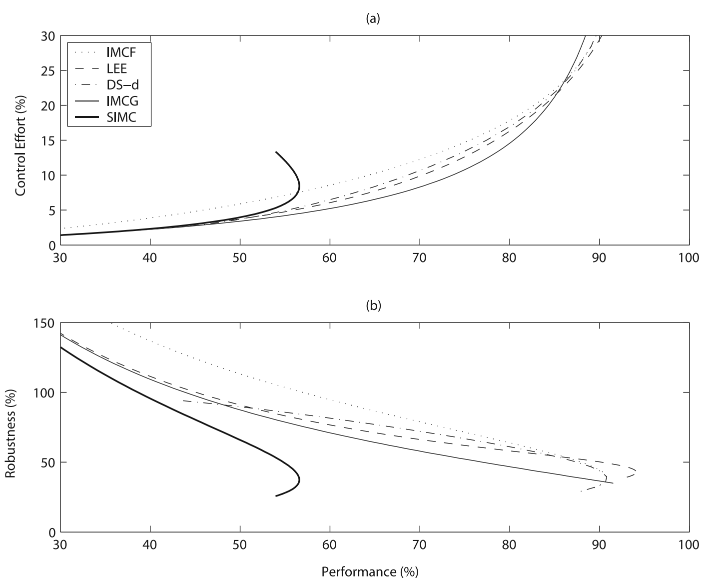

Figure 4. Parallel PID control of delay-dominant plant ($D/\tau = 2$, $\sigma_n^2 = 0$, $\alpha = 0.1$)

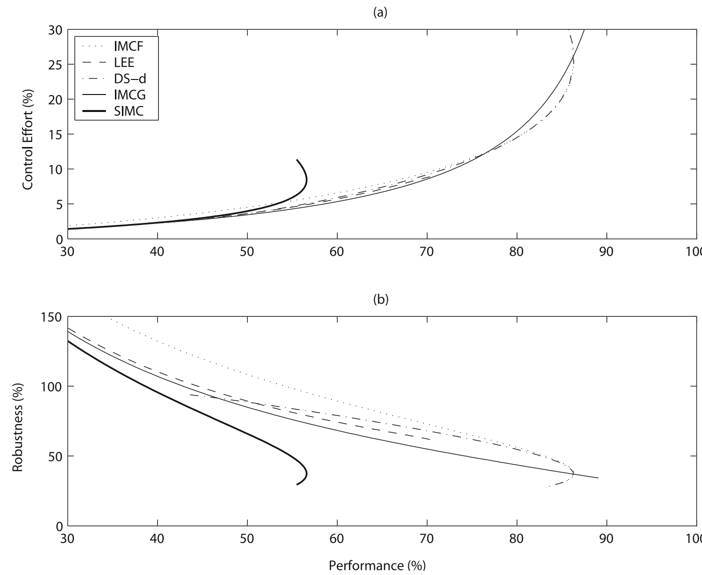

Figure 5. Series PID control of delay-dominant plant ($D/\tau = 2$, $\sigma_n^2 = 0$, $\alpha = 0.1$)

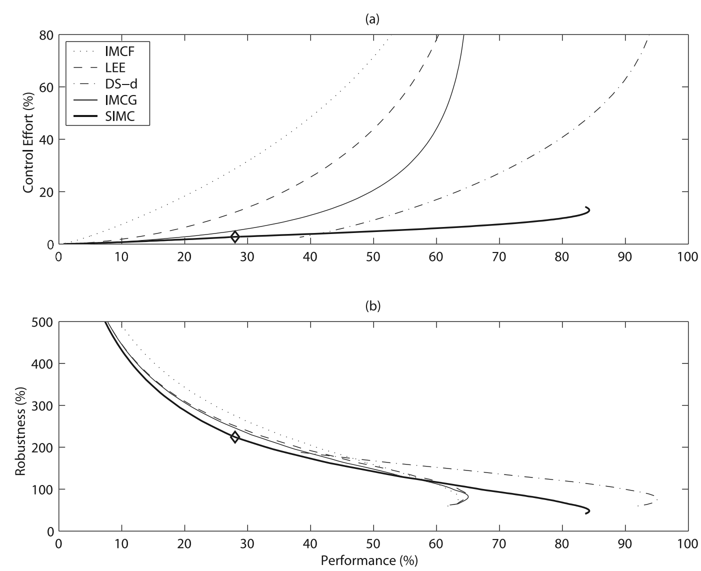

Figure 6. Parallel PID control of lag-dominant plant ($D/\tau = 0.1$, $\sigma_n^2 = 0.1$, $\alpha = 0.1$)

algorithm is superior to the others in terms of both robustness and control effort. (Notice, however, that the DS-d curve truncates when performance falls to 12% because additional increases in λ would send the derivative time below zero.) This scheme could be tuned for a control error variance only 17% greater than the benchmark parallel PID controller minimizing σ_e^2. As predicted in the Discussion, the IMCF strategy requires larger control effort than controllers tuned using the SIMC, DS-d and LEE methods. Virtually identical trends emerged in Figure 3 when the tuning rules were converted for the series form (5) using Equation (3.17) of Åström and Hägglund (1995). Based upon this and other case studies, it was concluded that differences between the series and parallel platforms are practically insignificant when the deadtime is small relative to the time constant (e.g. $D/\tau < 0.4$). Figure 4 displays the results obtained for the delay-dominant process

$$G(s) = \frac{e^{-20\,s}}{10\,s+1} \tag{41}$$

It is noteworthy that the PID design which requires the greatest control effort, namely IMCF, also possesses the greatest robustness. Conversely, the IMCG controller would make the smoothest valve adjustments but prove most sensitive to changes in the plant dynamics. This is counter-intuitive because one would naturally assume that if a controller is moving the manipulated variable in a more conservative manner than an alternate scheme tuned for the same performance, it would place less stringent requirements on model accuracy.

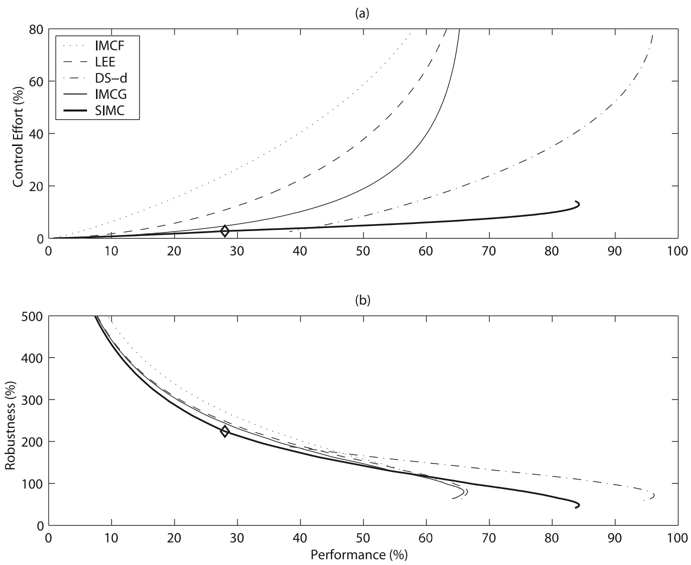

Figure 7. Series PID control of lag-dominant plant ($D/\tau = 0.1$, $\sigma_n^2 = 0.1$, $\alpha = 0.1$)

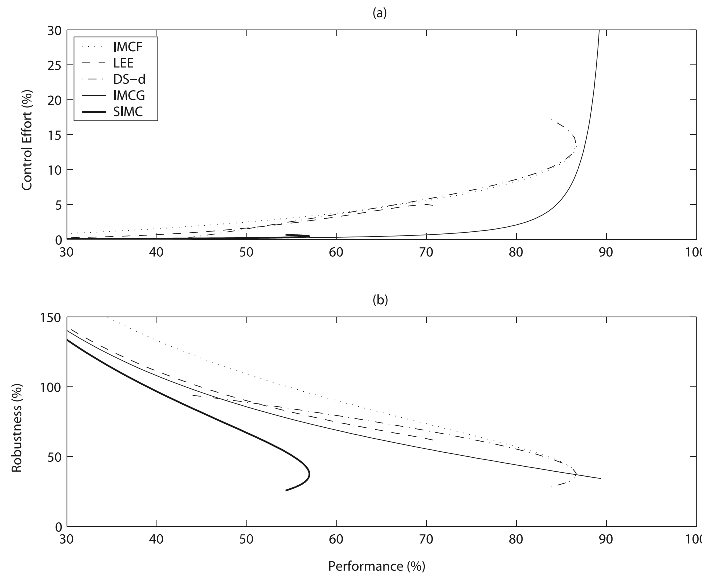

Figure 9. Series PID control of delay-dominant plant ($D/\tau = 2$, $\sigma_n^2 = 0.1$, $\alpha = 0.1$)

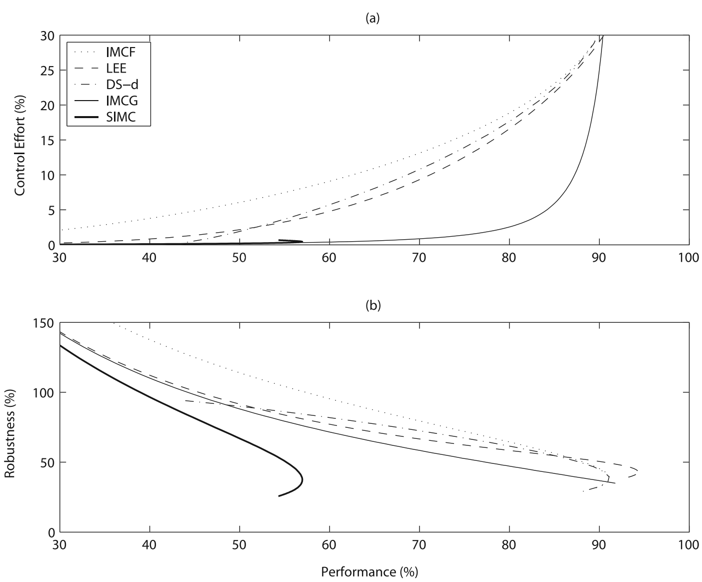

Figure 8. Parallel PID control of delay-dominant plant ($D/\tau = 2$, $\sigma_n^2 = 0.1$, $\alpha = 0.1$)

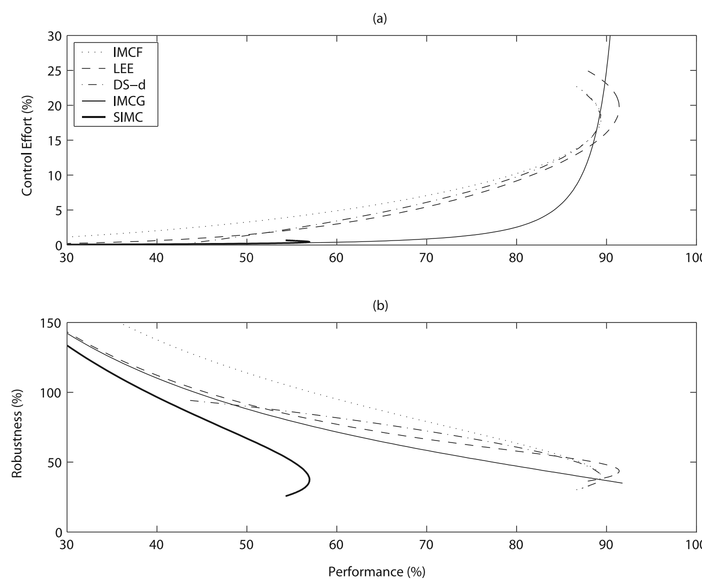

Figure 10. Parallel PID control of delay-dominant plant ($D/\tau = 2$, $\sigma_n^2 = 0.1$, $\alpha = 0.2$)

Implementing the IMCF and DS-d controllers via the series structure had very little effect on robustness but reduced their achievable performance by approximately 5% (see Figure 5). On the other hand, control effort was improved to about the same extent. In this example, the LEE strategy could not be simulated at performance levels exceeding 71% because the tuning formulae (17)-(19) then produced complex controller zeros. While the control effort of the SIMC PI algorithm compared favourably with that of the PID tuning methods, its robustness and achievable performance were obviously inferior. This is an illustration of the fact that incorporating the derivative mode usually enhances the performance and stability robustness of a feedback control loop, especially if D/τ is large. Provided that the process is free of measurement noise, these improvements can be derived with minimal rise in control effort. It is recommended that IMCF or IMCG be used for delay-dominant plants when the *PV* signal is smooth since both can be tuned for a wide range of performance under series and parallel PID control.

Figures 6 through 9 resulted when zero-mean, uncorrelated measurement noise of variance $\sigma_n^2 = 0.1$ was added to the process output. Figures 6 and 7 again show negligible difference between series and parallel control of (40). However, DS-d is no longer the obvious choice of tuning algorithm, unless one is only interested in near-minimum-variance operation. The achievable performance of the SIMC algorithm has risen from 47% in Figure 2 to 84% in Figure 6. This reflects the fact that there is less incentive for the use of derivative action in the control of lag-dominant processes when the measurement is corrupted by random sensor error. To understand why, recall from Figure 1 that the input disturbance enters through $G(s)$ which may be regarded as a low-pass filter. The controller design

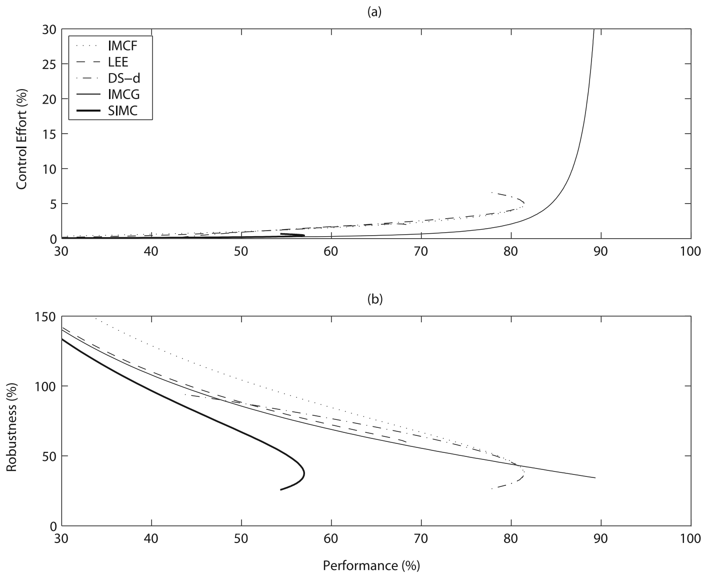

Figure 11. Series PID control of delay-dominant plant ($D/\tau = 2$, $\sigma_n^2 = 0.1$, $\alpha = 0.2$)

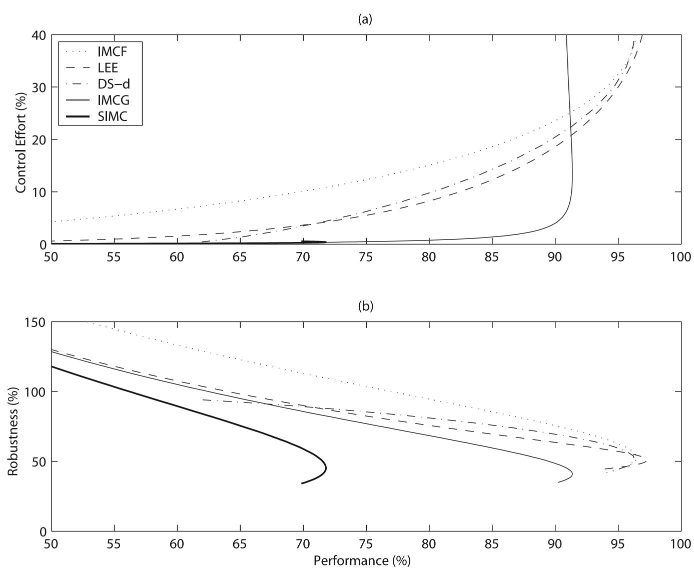

Figure 12. Parallel PID control of delay-dominant plant subject to output disturbance and measurement noise ($D/\tau = 2$, $\sigma_n^2 = 0.1$, $\alpha = 0.1$)

should therefore emphasize the elimination of low-frequency errors, rather than those present at high frequencies which will have been contributed mainly by the (uncontrollable) measurement noise. SIMC should be considered the default tuning method for noisy lag-dominant plants. This scheme may prove somewhat more sensitive to model/plant mismatch but, unlike DS-d, it can be detuned to an arbitrary extent and the variance of changes in the manipulated variable will usually be smaller.

For noisy systems with large D/τ, the SIMC control effort is often orders of magnitude less than that required by LEE, DS-d and IMCF (see Figure 8a). The main limitation of the SIMC approach is that its best achievable performance is under 60% for this example. The IMCG technique is capable of delivering high performance ($>$ 91%) and requires a much lower control effort than the three-parameter PID tuning methods, at least for performance levels up to around 90%.

Figure 9 illustrates that implementation of a PID controller using the series structure instead of the parallel platform generally implies, for the same value of α, significantly smoother valve adjustments at the expense of a decrease in achievable performance. This statement is supported as well by Figures 10 and 11. Upon comparing Figure 8 with Figure 10 and Figure 9 with Figure 11 it becomes evident that increasing α in either version of the PID transfer function has a similar effect; that is, to reduce both control effort and achievable performance. (The IMCG curves remained the same because $G_{C,3}$ was configured with $\alpha = 0.05$. Controller robustness does not appear to depend as strongly on the form of the PID regulator (series vs. parallel), nor upon variation in α.

When α in the parallel control system of Figure 8 is increased to around 0.25, the tuning curves obtained are practically identical to those displayed in Figure 9. Likewise, when α in the series PID loop of Figure 9 is reduced to 0.05, its response becomes very similar to that of the parallel controller with $\alpha = 0.1$. It may therefore be concluded that no significant difference would exist between the series and parallel implementations *in the absence of constraints upon* α, apart from the fact that a parallel PID controller cannot be translated to series form if it possesses complex zeros. (According to Åström and Hägglund (1995), however, the series algorithm is easier to tune by trial and error.)

As pointed out by Isaksson and Graebe (2002), derivative action would undoubtedly be more commonly employed if control engineers were afforded the same degree of flexibility when choosing α as in specification of the other tuning parameters. One major manufacturer of distributed control systems offers the choice between series PID with $\alpha = 0.1$ and a parallel controller with $\alpha = 1/16$. It is clear that users of this system should select the parallel (non-interacting) algorithm for high-performance applications, and the series (interacting) controller when variation of the manipulated variable is of greater concern than achieving near-minimum-variance regulation.

Output Disturbances

All of the case studies treated thus far have dealt with eliminating the effect of input disturbances on the process variable. Random walk or step disturbances entering at the process output are less frequently encountered in the process industries. Nevertheless, it is of interest to investigate the ability of the five design methods to handle such load disturbances. The tuning curves of Figure 12 are representative of the results obtained for a variety of noise levels and normalized deadtimes. These graphs are similar to the ones presented above for input disturbance rejection in delay-dominant systems (e.g. Figure 8). The controller with the greatest robustness (IMCF) exhibits highest control effort, and the PID algorithm with the worst robustness (IMCG) requires least control effort. The achievable performance and robustness of SIMC, which has no derivative mode, are markedly lower than those demonstrated by the PID controllers.

PID Control of Third-Order Plant

Finally, the sensitivity of these model-based tuning rules to the presence of unmodelled dynamics was studied by application to a third-order system based on Example 1.2.2 of Luyben and Luyben (1997). The unit comprises three well-stirred tanks in series. Heat is added to the first through a steam coil. The control

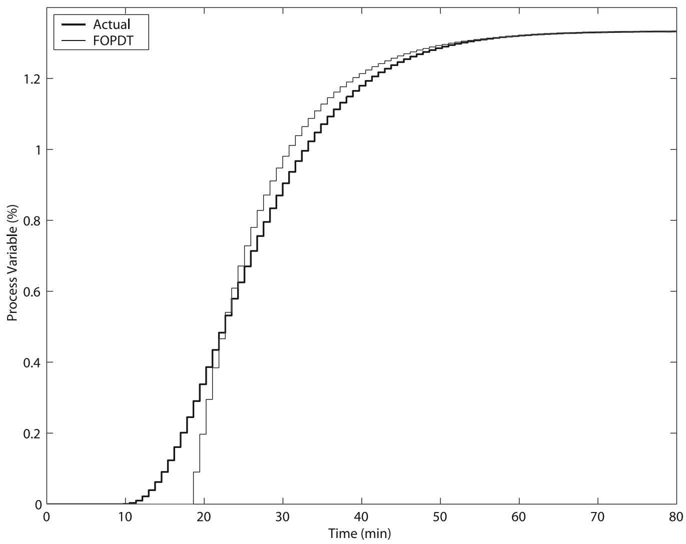

Figure 13. Open-loop response of measured Tank 3 temperature to 1% step increase in controller output

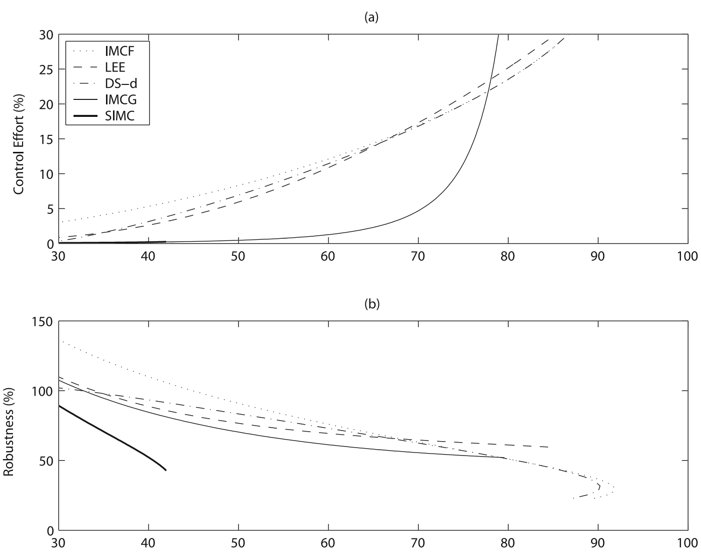

Figure 14. Parallel PID control of Tank 3 temperature ($D/\tau = 2$, $\sigma_n^2 = 0.178$, $\alpha = 0.1$)

objective is to regulate the temperature of the fluid leaving the third tank by manipulating the steam flow control valve position. The input disturbance d is (proportional to) the temperature of the cold feedstream entering the first tank and behaves as a random walk variable with $\sigma_a^2 = 1$. The process is subject to a transportation delay of nine minutes and the digital PID temperature controller employs a parallel algorithm with $\alpha = 0.1$.

A dynamic energy balance leads to the following plant transfer function

$$G_0(s) = \frac{1.33\, e^{-9s}}{(6s+1)^3} \tag{42}$$

which can be reduced to the FOPDT model

$$G(s) = \frac{K e^{-Ds}}{\tau s + 1} = \frac{1.33\, e^{-18s}}{9s+1} \tag{43}$$

using the ‘half-rule’ of Skogestad (2003). $G(s)$ is a delay-dominant process with $D/\tau = 2$. Its discrete-time step response is plotted against that of the actual plant $G_0(s)$ in Figure 13. The variance of the sensor noise $\sigma_n^2 = 0.178$ was chosen so that the ratio $\sigma_n^2/(K^2\sigma_a^2) = 0.1$, consistent with the value used in constructing Figure 8. Figure 14 reveals that the main effect of the model-order mismatch has been to reduce the robustness and achievable performance of the control strategies. It is interesting to note that the performance of IMCF, the algorithm which demonstrated greatest “throughput robustness” in Figure 8b, was least affected by the structural mismatch. The trends of Figures 8 and 14 are otherwise in reasonably good agreement, which suggests that the foregoing analysis will remain valid for higher-order systems when the open-loop dynamics can be well-approximated with a FOPDT model.

CONCLUSION

This paper has applied the methodology of Foley et al. (2005) to the analysis of model-based PID tuning strategies. The SIMC (Skogestad, 2003), DS-d (Chen and Seborg, 2002), IMC (Rivera et al., 1986) and Lee et al. (1998) tuning rules were presented for first-order-plus-deadtime processes and evaluated with respect to their robustness, control effort and achievable performance. The algorithms are easy to implement and possess a single tuning parameter (λ) which affects the closed-loop speed of response. The absence of such a feature in the Ziegler-Nichols and related methods explains to a large extent why they are infrequently employed in the chemical process industries. λ can be adjusted to obtain a satisfactory compromise between performance and control effort, or between performance and some measure of robust stability. It is chiefly through the selection of *tuning method* that one is able to strike a balance between robustness and control effort.

The simulation results were generally consistent with the “conventional wisdom” regarding use of the derivative mode. It was observed for systems which are free of measurement noise that the inclusion of derivative action improves the achievable performance and robustness at the expense of a moderate rise in control effort. However, when sensor noise is present, as is the case in the majority of industrial applications, then the benefits are usually only significant for delay-dominant plants. Unfortunately, the associated control effort may be prohibitively large, especially for techniques such as IMCF (Row F, Table 1 of Rivera et al., 1986), which place greater emphasis on high-frequency errors.

For a given level of performance, the magnitude of the control effort will depend on the form of the PID regulator (series vs. parallel) and its derivative filter factor, α. For lag-dominant processes ($D/\tau < 0.4$) it hardly matters which form is employed because there is little incentive for the use of derivative control. For delay-dominant systems, the series implementation of a PID tuning strategy will exhibit smoother valve adjustments than a parallel structure with the same value of α. The parallel format would permit better performance to be attained when minimizing process output variance is the primary goal. Increasing α in either version reduces both the control effort and best achievable performance. Most commercial control systems restrict α to a narrow range so the selection of PID structure for a delay-dominant application will be based on the relative importance of control effort and achievable performance. The IMCG scheme originally given in Row G of Table 1 by Rivera et al. (1986) was also considered. This approach relies on a third form of the PID controller which cannot be perfectly realized in many control systems. It was successfully approximated using series and parallel structures configured with minimum α and a first-order *PV* filter.

The correct choice of tuning strategy depends upon the process control objectives as well as the plant deadtime-to-time constant ratio. Except in the special case of lag-dominant systems with no measurement noise, where DS-d is clearly preferred, a trade-off emerges between control effort and robustness. For delay-dominant plants, the IMCF algorithm proved superior in robustness and IMCG in terms of control effort. Hence IMCG is recommended in situations where control effort should be minimized, e.g. to avoid interaction with other loops on a distillation column. Conversely, if the process dynamics are known to be nonlinear/time-varying and behaviour of the final control element is of little concern (e.g. in temperature control of a water-cooled chemical reactor), then IMCF will yield better results. When D/τ is small, it is proposed that SIMC tuning first be utilized to determine whether satisfactory performance can be obtained from this PI controller. If not, then one may resort to the DS-d scheme. SIMC will typically require less control effort than DS-d but it will also be less tolerant to errors in the process model. Future work will consider the implications of this research upon the selection of appropriate benchmarks for the assessment of PI(D) control loops.

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APPENDIX

The purpose of this section is to present intermediate calculations used to develop Figure 2. The z-transform of the lag-dominant process (40) is

$$G(z^{-1}) = z^{-4}\,\frac{(0.0315 + 9.68\times 10^{-4}\,z^{-1})}{1 - 0.968\,z^{-1}} \tag{A1}$$

when the sampling interval $T = 0.03(10 + 1) = 0.33$ time units. The function *fmincon* in the Matlab® Optimization Toolbox was invoked to minimize the control error variance

$$\sigma_e^2 = \frac{1}{2\pi j}\oint_{|z|=1} \frac{|G(z^{-1})|^2\,\sigma_a^2 + |1 - z^{-1}|^2\,\sigma_n^2}{|1 + G_{C,1}(z^{-1})G(z^{-1})|^2\,|1 - z^{-1}|^2}\,\frac{dz}{z} \tag{A2}$$

with $\sigma_n^2 = 0$, $\alpha = 0.05$ and $G_{C,1}(z^{-1})$ as defined by (37). The optimization was carried out subject to the constraints that the controller parameters were positive and that the feedback loop remained stable. This led to the optimal tuning constants

$K_C = 8.27$, $\tau_I = 1.55$, $\tau_D = 0.729$

and thus

$$G_{C,1}(z^{-1}) = \frac{27.4 - 43.7\,z^{-1} + 17.9\,z^{-2}}{1 - 1.10\,z^{-1} + 0.0995\,z^{-2}}$$

$$\Rightarrow\ \sigma_{e,MV}^2 = 0.0712,\quad \sigma_{\Delta U,MV}^2 = 2.27$$

using Equations (37)-(39).

When λ is specified as 0.7 and the deadtime D in (12)-(14) is inflated to $1 + 0.33/2 = 1.17$, the IMCF design equations yield

$K_C = 8.25$, $\tau_I = 10.6$, $\tau_D = 0.550$

Evaluating (38) and (39) for the parallel implementation with $\alpha = 0.1$ gives

$\sigma_e^2 = 0.237$, $\sigma_{\Delta U}^2 = 0.904$

which implies a performance of $0.0712/0.237 \times 100\% = 30.0\%$. and control effort $0.904/2.27 \times 100\% = 39.8\%$.An interval-halving procedure was then applied to locate the maximum parameter error δ (cf. Equation (9)) for which all roots of the characteristic equation $1 + G_{C,1}(z^{-1})G_0(z^{-1}) = 0$ lay within the unit circle $|z| = 1$. The robustness, $\delta_{max,}$ of the IMCF controller

Table A1. Parallel PID control of lag-dominant plant
(*Performance* $= 30\%$, $D/\tau = 0.1$, $\sigma_n^2 = 0$)

Method	K_C	τ_I	τ_D	α	τ_F	Control Effort (%)	Robustness (%)
IMCF	8.25	10.6	0.550	0.1	N/A	39.8	84.7
LEE	8.24	10.5	0.511	0.1	N/A	35.7	86.5
DS-d	5.61	4.46	0.328	0.1	N/A	12.0	129
IMCG	8.29	10.6	0.550	0.05	0.0507	35.1	83.8
SIMC	6.70	5.97	N/A	N/A	N/A	14.2	85.8

$$G_{C,1}(z^{-1}) = \frac{20.3 - 33.1\,z^{-1} + 13.0\,z^{-2}}{1 - 1.14\,z^{-1} + 0.143\,z^{-2}}$$

was found to be 84.7%. Table A1 lists the corresponding results when the other methods were tuned for the same level of performance.

NOMENCLATURE

d	input disturbance
D	nominal process deadtime
DS-d	Direct Synthesis for Disturbance Rejection
e	control error, r - Y
FOPDT	First-Order-Plus-Deadtime
$G(s)$	process transfer function
$G_C(s)$	feedback controller transfer function
$G_C^*(s)$	Internal Model Control law
$G_{C,r}(s)$	servo compensator
IMC	Internal Model Control
IMCF	tuning rule given in Table 1, Row F of Rivera et al. (1986)
IMCG	tuning rule given in Table 1, Row G of Rivera et al. (1986)
K	steady-state gain of nominal process
K_C	proportional gain of parallel PID controller
K_C'	proportional gain of series PID controller
LEE	tuning method presented as Equations (22) of Lee et al. (1998)

n	measurement noise
PI	Proportional-Integral
PID	Proportional-Integral-Derivative
PV	Process Variable
r	setpoint
s	Laplace transform variable
SIMC	Skogestad IMC
T	sampling/control interval
U	controller output (%)
Y	process variable, i.e., measurement of controlled variable (%)
z^{-1}	backshift operator

Greek Symbols

α	derivative filter factor
δ	correlated parameter estimation error
Δ	differencing operator, $1 - z^{-1}$
λ	adjustable parameter in PID tuning algorithms
Λ	normalized tuning parameter, λ/τ
σ_a^2	variance of differenced input disturbance, Δd_t
σ_e^2	control error variance
$\sigma_{e,MV}^2$	minimum error variance achievable using parallel PID control with $\alpha = 0.05$
σ_n^2	variance of measurement noise
$\sigma_{\Delta U}^2$	variance of adjustments in manipulated variable
$\sigma_{\Delta U,MV}^2$	variance of control moves exhibited by minimum variance parallel PID controller with $\alpha = 0.05$
τ	nominal process time constant
τ_D	derivative time of parallel PID controller
τ'_D	derivative time of series PID controller
τ_F	PV filter time constant
τ_I	integral time of parallel PID controller
τ'_I	integral time of series PID controller
Ψ	normalized deadtime, D/τ

Subscripts

max	maximum value for which closed loop remains stable
0	actual value
1	parallel form of the PID controller
2	series form of the PID controller
3	"textbook" PID in series with first-order filter

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